1.

$$
\begin{aligned}
& H(AB) - H(A) - H(B|A) \\
= \; & -\sum_i \sum_j p(A_i, B_j) + \sum_i p(A_i) \log p(A_i) + \sum_i p(A_i) H(B|A_i) \\
= \; & \sum_i p(A_i) \log p(A_i) + \sum_i \sum_j p(A_i, B_j) \log p(B_j|A_i) - \sum_i \sum_j p(A_i, B_j) \log p(A_i, B_j) \\
= \; & \sum_i p(A_i) \log p(A_i) + \sum_i \sum_j p(A_i, B_j) \log \frac{p(B_j|A_i)}{p(A_i, B_j)} \\
= \; & \sum_i p(A_i) \log p(A_i) + \sum_i \sum_j p(A_i, B_j) \log \frac{1}{p(A_i)} \\
= \; & \sum_i p(A_i) \log p(A_i) - \sum_i p(A_i) \log p(A_i) \\
= \; & 0
\end{aligned}
$$

2.

$$
\begin{aligned}
I(A;B) &= H(A) - H(A|B) \\
&= H(AB) - H(B|A) - H(A|B) \\
&= HB() - H(B|A) \\
&= I(B;A)
\end{aligned}
$$

3.

$$
\begin{aligned}
H(AB) &\leq H(A) + H(B) \text{ より}, \\
H(AB) - H(A) &\leq H(B) \\
H(B|A) &\leq H(B) \\
\text{また}, H(AB) - H(B) &= H(A|B) \geq 0 //
\end{aligned}
$$

4.

$$
\begin{aligned}
I(A;B) = H(A) - H(A|B) \geq 0 \\
H(A) - I(A;B) = H(A|B) \geq 0
\end{aligned}
$$