片方だけ

- 冪等律

$$a \vee a = \sup\{a, a\} = a$$

- 交換律

$$a \vee b = \sup\{a, b\} = \sup\{b, a\} = b \vee a$$

- 結合律

  $(a \vee b) \vee c = \sup\{\sup\{a, b\}, c\}$ は $\{a, b, c\}$ の上界で, $\sup\{a, b, c\}$ は, その最小元だから,

$$\sup\{\sup\{a, b\}, c\} \quad \succeq \quad \sup\{a, b, c\} \tag{1}$$

  また, $\sup\{a, b, c\}$ は $\{a, b\}$ の上界で, $\sup\{a, b\}$ がその最小元だから

$$\sup\{a, b\} \quad \preceq \quad \sup\{a, b, c\}$$

  これと, $c \preceq \sup\{a, b, c\}$ より,

$$\begin{aligned} \sup\{\sup\{a, b\}, c\} &\preceq \sup\{a, b, c\} && (2) \\ (1), (2)\text{ より}, \sup\{\sup\{a, b\}, c\} &= \sup\{a, b, c\} \\ \text{対称性より}, \sup\{a, \sup\{b, c\}\} &= \sup\{a, b, c\} \\ \text{ゆえに}, \sup\{\sup\{a, b\}, c\} &= \sup\{a, \sup\{b, c\}\} \\ \text{つまり}, (a \vee b) \vee c &= a \vee (b \vee c) \end{aligned}$$

- 吸収律

$$\begin{aligned} a \wedge (a \vee b) = \inf\{a, \sup\{a, b\}\} \\ \text{ここで}, \sup\{a, b\} = x \text{ とおくと}, a \preceq x \\ \text{ゆえに}, a \wedge (a \vee b) = \inf\{a, x\} = a \end{aligned}$$