

(1)

1. • 冪等律

$$A \cap A = A$$

$$A \cup A = A$$

- 交換律

$$A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

- 結合律

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

- 吸収律

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

2. • 双対的分配律

$$\begin{aligned} (A \cap B) \cup (A \cap C) &= ((A \cap B) \cup A) \cap ((A \cap B) \cup C) \quad (\text{分配律}) \\ &= A \cap ((A \cap B) \cup C) \quad (\text{吸収律}) \\ &= A \cap ((A \cup C) \cap (B \cup C)) \quad (\text{分配律}) \\ &= A \cap (B \cup C) \quad (\text{結合律, 吸収率}) \end{aligned}$$

- モジュラ律

$$A \cup B = B \quad (\text{仮定})$$

$$(A \cup B) \cap (A \cup C) = B \cap (A \cup C) \quad (\cap(A \cup C))$$

$$A \cup (B \cap C) = (A \cup C) \cap B \quad (\text{分配律, 交換律})$$

(2)

$$x \vee y = x \vee z (= M)$$

$$y \wedge (x \vee y) = y \wedge (x \vee z)$$

$$y = (y \wedge x) \vee (y \wedge z)$$

$$= m \vee (y \wedge z)$$

$$= y \wedge z$$

対称性より,  $z = y \wedge z$  なので,  $y = z$